General Linear Model for Longitudinal Data

MLE estimation for :

The likelihood function is:

In order to obtain MLE, we can take the derivative with respective to these two parameters and set them to be zero:

MLE Algorithm:

Start with some initial values of (for example, )

For t in 1:T

Estimate   based on

Estimate   based on

Until convergence

Properties of MLE:

Under regulatory conditions,   and are consistent, and

Where . There, the asymptotic variance of is , which is consistently estimated by

So that

REML estimation:

So in some sense,

Where

For REML estimation, we need to iterate the estimation procedure, and is more complicated than the MLE estimation

In certain settings, is unbiased for (unlike ). This is not always the case. However, REML often has smaller bias and MSE than MLE estimates.

Finally,